EXTENSION OF THE TABLE FOR THE CALCULATION OF SURFACE TENSION FROM MEASUREMENTS OF SESSILE DROPS

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ABSTRACT. The calculations presented are an extension of the work of Tawde and Parvatikar on sessile drops. A table is prepared of the values of a^2/r^2 against h/r in the range from 0 570 to 0 978 at interval of 0.001.

The use of sessile drops for the measurement of surface tension has been suggested by several workers such as Worthington (1885), Fergusson (1913), Rayleigh (1915) and Porter (1933), but it is only recently that the method was placed on better foundations for exact work by Taylor and Alexander (1944). These authors have fitted up an empirical equation from which the values of a^2/r^2 are obtained for the corresponding values of h/r where a^2 is the capillary constant which connects the surface tension γ of a liquid by the relation $\frac{2\gamma}{g\rho}$, ρ being the effective density of the liquid, r, the radius of the sessile drop and h, the height of it from the equatorial plane. Thus the final derivation of surface tension from the knowledge of the measurable quantity h/r involves the use of the tabular functions connecting h/r with a^2/r^2.

Recently, Tawde and the author (1951) have shown that this function of h/r and a^2/r^2 could also be obtained by modifying the standard tables of Bashforth and Adams (1883). The table thus drawn up from fundamental considerations has been put to a rigorous test for its usefulness by using the experimental measurements on sessile drops. By a critical study, it has been shown that this table is equally dependable for applicability to experimental measurements. This table gave values of h/r and a^2/r^2 for the values of β ($\beta = 2b^2/a^2$, where b is the radius of curvature at the apex of the drop) ranging from 25 to 50 at interval of unity and from 50 to 100 at interval of two. Later, Tawde and Parvatikar (1954) have prepared a more detailed and conprehensive table of h/r *vs.* a^2/r^2, but, however, only in the range from 0.5100 to 0.5708.

Now it is interesting to consider whether it is necessary to extend this table beyond the range for which it was worked out. It can easily be seen that for a given area of a flat circular tip, the drops of a liquid formed on the tip will not have diameters beyond a certain limit. Thus the range of the shape factor h/r of a

TABLE I

h/r vs a^2/r^2 for sessile drops.

h/r	0	1	2	3	4	5	6	7	8	9
0.57	0.36393	0.36589	0.36784	0.36984	0.37182	0.37382	0.37581	0.37785	0.37988	0.38196
0.58	0.38403	0.38609	0.38813	0.39028	0.39238	0.39454	0.39665	0.39876	0.40091	0.40312
0.59	0.40530	0.40748	0.40968	0.41192	0.41412	0.41637	0.41865	0.42090	0.42318	0.42550
0.60	0.42779	0.43012	0.43249	0.43484	0.43721	0.43960	0.44198	0.44443	0.44687	0.44928
0.61	0.45173	0.45424	0.45675	0.45926	0.46178	0.46434	0.46689	0.46945	0.47202	0.47460
0.62	0.47721	0.47986	0.48252	0.48517	0.48786	0.49056	0.49328	0.49601	0.49876	0.50155
0.63	0.50434	0.50715	0.50994	0.51279	0.51565	0.51854	0.52145	0.52434	0.52723	0.53017
0.64	0.53313	0.53613	0.53915	0.54219	0.54524	0.54830	0.55134	0.55444	0.55759	0.56074
0.65	0.56388	0.56706	0.57027	0.57353	0.57680	0.58003	0.58335	0.58670	0.59005	0.59342
0.66	0.59683	0.60024	0.60366	0.60711	0.61056	0.61405	0.61758	0.62112	0.62466	0.62828
0.67	0.63193	0.63561	0.63929	0.64302	0.64675	0.65051	0.65428	0.65805	0.66188	0.66575
0.68	0.66965	0.67355	0.67749	0.68149	0.68551	0.68957	0.69366	0.69774	0.70184	0.70597
0.69	0.71014	0.71434	0.71858	0.72284	0.72717	0.73150	0.73583	0.74024	0.74467	0.74915

TABLE I (*contd.*)

h/r vs. a^2/r^2 for sessile drops.

h/r	0	1	2	3	4	5	6	7	8	9
0.70	0.75365	0.75815	0.76272	0.76737	0.77202	0.77666	0.78134	0.78603	0.79081	0.79560
0.71	0.80047	0.80535	0.81026	0.81518	0.82020	0.82525	0.83036	0.83550	0.84066	0.84584
0.72	0.85106	0.85637	0.86169	0.86711	0.87252	0.87798	0.88344	0.88898	0.89457	0.90018
0.73	0.90587	0.91158	0.91735	0.92315	0.92897	0.93492	0.94087	0.94689	0.95297	0.95904
0.74	0.96522	0.97140	0.97763	0.98398	0.99033	0.99679	1.0033	1.0098	1.0164	1.0231
0.75	1.0298	1.0366	1.0434	1.0502	1.0572	1.0642	1.0713	1.0784	1.0857	1.0929
0.76	1.1003	1.1077	1.1152	1.1227	1.1304	1.1381	1.1458	1.1536	1.1615	1.1695
0.77	1.1774	1.1856	1.1938	1.2020	1.2104	1.2188	1.2273	1.2358	1.2445	1.2534
0.78	1.2622	1.2711	1.2801	1.2893	1.2985	1.3077	1.3171	1.3267	1.3363	1.3458
0.79	1.3556	1.3655	1.3754	1.3853	1.3953	1.4058	1.4163	1.4268	1.4372	1.4480
0.80	1.4589	1.4699	1.4808	1.4918	1.5032	1.5148	1.5264	1.5380	1.5495	1.5617
0.81	1.5739	1.5861	1.5983	1.6105	1.6232	1.6362	1.6492	1.6622	1.6751	1.6883
0.82	1.7021	1.7159	1.7297	1.7435	1.7573	1.7717	1.7865	1.8014	1.8162	1.8311
0.83	1.8459	1.8614	1.8774	1.8934	1.9094	1.9254	1.9414	1.9577	1.9751	1.9925
0.84	2.0099	2.0273	2.0447	2.0621	2.0804	2.0995	2.1185	2.1376	2.1567	2.1757

TABLE I (*contd.*)

h/r vs. a^2/r^2 for sessile drops.

h/r	0	1	2	3	4	5	6	7	8	9
0.85	2.1948	2.2148	2.2359	2.2570	2.2780	2.2991	2.3202	2.3412	2.3623	2.3858
0.86	2.4093	2.4329	2.4564	2.4800	2.5035	2.5271	2.5506	2.5773	2.6030	2.6318
0.87	2.6576	2.6843	2.7111	2.7379	2.7647	2.7930	2.8241	2.8552	2.8863	2.9174
0.88	2.9485	2.9796	3.0107	3.0417	3.0737	3.1105	3.1472	3.1839	3.2206	3.2573
0.89	3.2940	3.3308	3.3675	3.4042	3.4425	3.4874	3.5323	3.5772	3.6220	3.6669
0.90	3.7118	3.7567	3.8016	3.8465	3.8914	3.9409	3.9981	4.0553	4.1126	4.1698
0.91	4.2270	4.2843	4.3415	4.3987	4.4560	4.5132	4.5704	4.6387	4.7164	4.7941
0.92	4.8717	4.9494	5.0271	5.1048	5.1825	4.2601	5.3378	5.4155	5.4932	5.5709
0.93	5.6686	5.7849	5.9012	6.0174	6.1337	6.2500	6.3662	6.4825	6.5988	6.7150
0.94	6.8313	6.9476	7.0638	7.1801	7.3036	7.5096	7.7157	7.9217	8.1277	8.3338
0.95	8.5398	8.7359	8.9519	9.1580	9.3640	9.5700	9.7761	9.9822	10.188	10.403
0.96	10.600	11.087	11.692	12.170	12.711	13.253	13.794	14.335	14.876	15.418
0.97	15.959	16.505	17.041	17.583	18.124	18.665	19.207	19.748	20.289	—

liquid is governed by the tip used. This range can be easily altered by employing a different tip Therefore, if the table of h/r *vs.* a^2/r^2 is limited in its range a suitable tip will have to be chosen so as to form drops of a liquid having h/r within this range.

It can be seen from the observations of Taylor and Alexander (1944) that this shape factor h/r is within the range 0 46505 and 0.48462, whereas this range is between 0.5203 and 0 5539 for the observations of Tawde and the author (1951) This shift of range of h/r is to be attributed to the different tip employed for the formation of drops. But it is a point to be noted here that r or h/r does not vary much from each other in the two sets of observations This is to be expected due to the circular tips employed in the two independent investigations having nearly the same diameter. Unfortunately, since the diameter of the flat circular tip used has not been given by Taylor and Alexander (1944) in their paper, it is not possible to verify the above conclusion But the experience of the author on similar experimental investigations shows that the conclusions drawn may not be far from truth. Therefore, if tips selected vary widely, it is possible to produce drops of liquids having values of h/r in different regions The standard tables of Bashforth and Adams (1883) in the case of sessile drops are at intervals 0.1 in β in the range 0.0 to 46.7 As pointed out earlier the table of h/r *vs.* a^2/r^2 has been worked out only in the range of $h/r = 0.5100$ to 0.5708 (or $\beta = 22.0$ to 46.7), and therefore, it is thought desirable to extend this table for the remaining values of β available in the work of Bashforth and Adams This has been done and a table is drawn in the range of $\beta = 0$ 0 to 22.0 The method of calculation to obtain the values of h/r and a^2/r^2 was the same as shown in the earlier work

Direct interpolation was used to arrive at a^2/r^2 corresponding to the desired value of h/r The table thus prepared is given below with h/r values at interval of 0.001. This table allows for direct interpolation of intermediate values of h/r

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